Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2022

Subject Name: Differential Equations

Subject Code: 5SC01DIE1 **Branch: M.Sc. (Mathematics)**

Semester:1 **Date: 22/04/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$. (02)

b. State Legendre's differential equation (01)

c. Check whether $\{1, e^x, e^{-x}\}$ linearly independent or dependent. (02)

d. Determine the nature of the point $x = 0$ for the equation, (02)

$xy'' + y \sin x = 0.$

Q-2 **Attempt all questions** **(14)**

a. Solve the ordinary differential equation $y'' - 3y' + 2y = e^x$by the method of Variation of parameters. (05)

b. Determine the series solution of the differential equation $y'' - y = 0$ at $x = 0$. (05)

c. Solve:$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x.$ (04)

OR

Q-2 **Attempt all questions** **(14)**

a. Find the series solution of $(x - 1)y'' + xy' + y = 0$ with $y(0) = 2$, $y'(0) = -1$. (07)

b. By power series method solve the initial value problem $xy'' + y' + 2y = 0$ with $y(1) = 2, y'(1) = 4$. (07)

Q-3 **Attempt all questions** **(14)**

a. State and prove Orthogonality of Legendre's polynomials. (07)

b. State and Prove Rodrigue's Formula. (07)

OR

Q-3 Attempt all questions (14)

a. Prove that $(1-2xt+t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} t^n P_n(x)$, where $|x| \le 1, |t| < 1$ (06)

b. In usual notation prove that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)d\theta$ if $m = n$. (06)

c. Show that $P_n(1) = 1$. (02)

SECTION – II

Q-4 Attempt the Following questions (07)

a. Find $\Gamma(\frac{1}{2})\,\Gamma(\frac{5}{2})$. (02)

b. $J_{\frac{1}{2}}(x) =$ _________ (01)

c. Find complete integral of $p + q + pq = 0$. (02)

d. State Lagrange's equation. (01)

e. Write formula of $\Gamma(n+1)$. (01)

Q-5 Attempt all questions (14)

a. State and prove Fourier Bessel's expansion of $f(x)$. (06)

b. Show that $J_n(ax)$ is solution of $y'' + \frac{1}{x}y' + \left(a^2 - \frac{p^2}{x^2}\right)y = 0$ (05)

C. Express $x^2 - 3x + 1$ in terms of Legendre polynomials. (03)

OR

Q-5 Attempt all questions (14)

a. Form partial differential equation by eliminate arbitrary function from following (05)

(a) $z = e^y f(x+y)$

$(b) z = f(x^2 - y^2)$.

b. Find the third approximation of the solution of the equation $\frac{dy}{dx} = 2 - \left(\frac{y}{x}\right)$, $y(1) = 0$ by Picard's method (05)

c. Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$. (04)

Q-6 Attempt all questions (14)

a. Solve the following partial differential equations. (06)

(a) $y^2p - xyq = x(z - 2y)$.

(b) $z(xp - yq) = y^2 - x^2$

b. Find complete integral of $z + 2u_z - (u_x + u_y)^2 = 0$ by Jacobi's method. (06)

c. Prove that $J_n(x) = (-1)^n J_{-n}(x)$. (02)

OR

Q-6 **Attempt all Questions** **(14)**

a. If $\bar{X}\ curl\ \bar{X} = 0$, where $\bar{X} = (P, Q, R)$ and μ is an arbitrary differential function of x, y and z then prove that $\mu\,\bar{X}\ \cdot curl\ (\mu\,\bar{X}\,) = 0$. (06)

b. Verify that the Pfaffian differential equation $yz\ dx + xzdy + xy\ dz = 0$ is integrable and find its solution. (05)

c. Check whether the partial differential equations $p = 1 + e^{\frac{x}{y}}$ and $q = e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)$ are compatible or not. (03)